Radiative Convective Equilibrium, the Greenhouse Effect and Cloud Radiative Effects

Weston Anderson

December 19, 2018

Contents

1 Introduction

In these notes we'll discuss the energy balance of earth. In general, energy can be transferred around a system (such as the earth) in three ways:

1.Radiation - Energy moving through space requiring neither a medium to travel through nor any exchange of mass. This is how the energy from the sun reaches the earth, traveling at the speed of light.

2. Convection - Transfer of energy through an exchange of mass. In the atmosphere parcels of air with different energy levels will change places and transfer energy around the system.

3. Conduction - No mass is exchanged, but energy is transferred through a medium by collisions between atoms or molecules.

2 Basics of radiation

The sun emits a near constant amount of energy (solar luminosity) equal to $L_0 = 3.9x10^26W$. Because space is a vacuum, we can assume that the total amount of energy passing through a sphere of any size with the sun at its center will remain constant (and equal to L_0). However, the density of this flux (S_d) will change with increasing distance from the sun (d) as:

$$Flux = L_0 = 4S_d\pi d^2$$

We often use the above equation along with the mean distance of the earth from the sun to estimate the flux of energy reaching the earth from the sun (the 'solar constant', $S_0 = 1367\ W/m^2$).

Our starting point for discussing the radiative balance of the earth will be blackbody radiation, which is the radiation field of an object with unit emissivity (i.e. the object absorbs and re-emits a maximum amount of energy). Blackbody radiation refers to a continuous spectrum of wavelengths, and depends only on temperature as described by the Stefan-Boltzmann Law:

$$E_{BB} = \sigma T^4$$

where σ is the Boltzmann constant ($\sigma = 5.67 \times 10^{-8}\ \frac{W}{m^2K^4}$). Because most objects are not blackbodies, we will define the emissivity, ϵ, as the ratio of actual emission of a body or volume of gas compared to a blackbody at the same temperature. This means that the equilibrium emission of an object follows

$$E_R = \epsilon\sigma T^4$$

We can further define the peak of a blackbody emission spectrum for a given temperature by following Wein's law:

$$\lambda = \frac{b}{T}$$

where b is a constant. From this we can deduce that warmer objects (like the sun) emit higher frequencies and shorter wavelengths than relatively cool objects (like the earth). We'll return to this concept throughout these notes.

3 Emission temperature of earth

3.1 Radiative equilibrium

Now that we understand the basics of radiation reaching the earth, we'll discuss how this radiation makes it's way through the earth system before being re-emitted to space. In the most fundamental sense, our starting point for an energy budget of earth is that (Solar radiation absorbed) = (planetary radiation emitted). So let us begin by defining the solar radiation absorbed by the earth. Because the earths radius is small compared to its distance from the sun, we can approximate incoming solar radiation as a series of parallel and uniform beams. The energy intercepted by the surface of the earth may then be thought of as equivalent to the shadow that would be cast by an object in the beam of a flashlight (see Fig. 1).

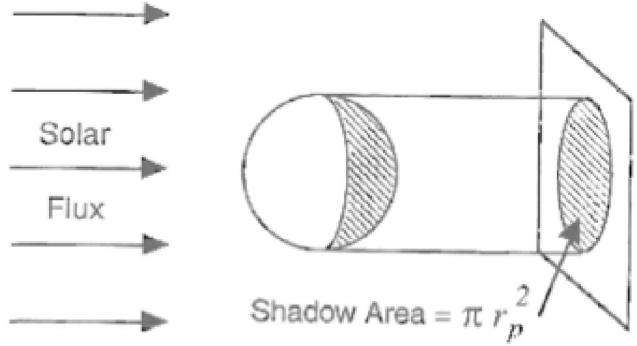

Fig. 2.2 Diagram showing the shadow area of a spherical planet.

Figure 1: Credit: Figure 2.2 of (**?**)

We now need to account for the fact that not all of the incoming solar radiation is absorbed by earth, some is reflected back into space. We refer to the reflectivity of earth as its 'albedo' (α), and generally use a globally averaged value of 0.3 (meaning the earth absorbs 70% of incident radiation). This makes the total absorbed solar radiation:

Solar radiation absorbed $= S_0(1-\alpha)\pi r_p^2$

Now because the entire surface of the earth will re-emit the absorbed radiation, we are no longer considering the 'shadow area' of the earth but the surface area of a sphere.

Emitted terrestrial radiation $= 4\sigma T_e^4 \pi r_p^2$

From this equation we can set the solar radiation absorbed equal to the emitted terrestrial radiation and solve for the terrestrial emitting temperature (surface temperature). But this gives us a value of 255 K ($-18°$C), which is far too cold to be the average surface temperature of the earth! To understand the difference between the observed emission temperature and our calculated emission temperature we need to consider the greenhouse effect.

4 Greenhouse effect

We will begin exploring the greenhouse effect by adding a very simple atmosphere into our model, one that is transparent to incoming solar radiation, but acts as a blackbody for emitted terrestrial radiation (see below diagram). The atmosphere emits both back to the surface of the earth and out into space. In this model the only radiation emitted to space is from the atmosphere. We can further see that the presence of an atmosphere will warm the surface, because all of the solar insolation is still reaching the surface of the earth, but there is now an additional term due to downward emission of radiation from the atmosphere.

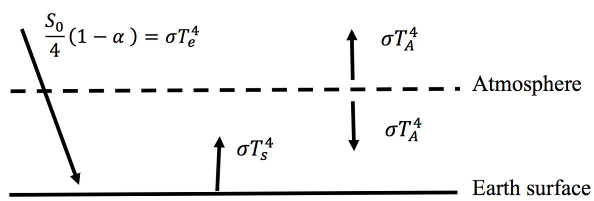

Figure 2: Energy budget for earth using a single layer atmosphere transparent to incoming solar radiation but which acts as a blackbody for terrestrial radiation

In the above figure we've further introduced the concept of an effective emitting temperature of the earth (T_e). In the absence of an atmosphere this was simply the surface temperature. But now because there are additional terms, we need to be more precise. Here we can say that, due to conservation of energy, the emitting temperature can be calculate as the equivalent blackbody radiation.

In reality the atmosphere does not act as a blackbody, but rather absorbs only a portion of incoming terrestrial radiation before reemitting it in both directions. We can add this into our model as an emissivity parameter, allowing the radiation that is not absorbed pass through to be emitted to space.

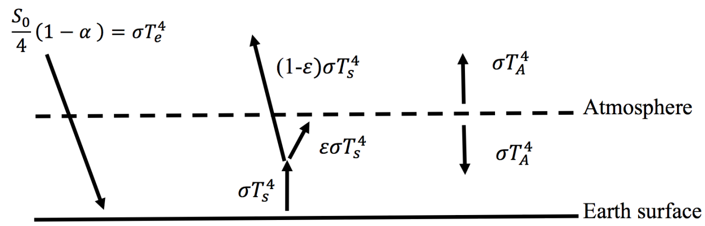

Figure 3: Energy budget for earth using a single layer atmosphere transparent to incoming solar radiation and with an emissivity parameter for terrestrial radiation

By introducing this concept we can consider two limits:

1. Large ϵ - The atmosphere is optically thick (if $\epsilon = 1$, the atmosphere is a blackbody). In this limit longwave emission to space is coming from the atmosphere entirely
2. Small ϵ - The atmosphere is optically thin. Emission to space is coming mainly from the surface.

The diagram below illustrates the full solution between these two limits. As longwave opacitiy of the atmosphere increases, the outgoing flux increasingly comes from the atmosphere, where the surface emission is absorbed. At the surface, the emission (and temperature) rise with increasing atmospheric opacity to account for in increasing amount of radiation re-emitted to the surface from the atmosphere. This is the greenhouse effect.

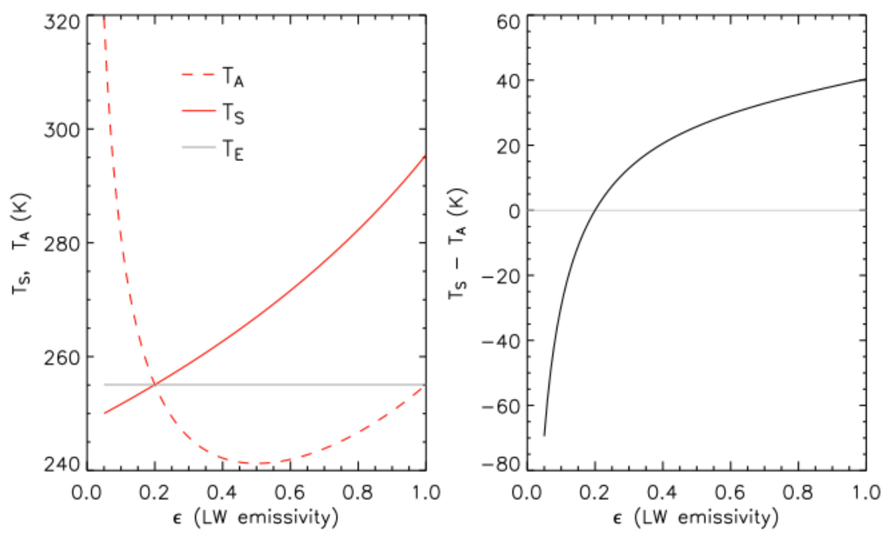

Figure 4: Figure credit: notes of Ron Miller, NASA GISS

4.1 Radiative convective equilibrium

In radiative equilibrium we assume that no convection takes place, and so that every vertical layer of the atmosphere is individually in equilibrium. In radiative convective equilibrium we begin to introduce concepts of atmospheric dynamics by allowing 1-D convection to take place within the vertical column of the atmosphere. To understand when this will occur, let us return to our two layer radiative equilibrium model.

4.1.1 Two layer model

In a simple 1-D model, the temperature difference between the atmosphere and the surface is a measure of the stability of the air column. If the surface is cooler than the atmosphere above it, the column is stable and no convection is necessary. If, however, the temperature of the surface is significantly warmer than the air above it, then the column is said to be unstable because warm air tends to rise. We can therefore introduce a measure of the stability of an air column as the difference in temperatures per unit height, or a 'lapse rate': $\gamma = \frac{dT}{dz}$.

We can further estimate at what lapse rate the atmospheric column is stable by assuming that a parcel of air rises adiabatically – that is, without transferring energy to its surroundings – but that it cools as it rises. We begin with the first law of thermodynamics:

$$Q = 0 = C_p dT - \frac{dP}{\rho}$$

where Q is the heating or cooling (Q=0 because the motions are adiabatic) and C_p is the heat capacity of an ideal gas at constant pressure. We can then use the hydrostatic balance ($\frac{dP}{dz} = -\rho g$) to subsititute in for dP to get:

$$Q = 0 = C_p dT - \frac{-\rho g dz}{\rho}$$

or, equivalently

$$\frac{dT}{dz} = -\frac{g}{C_p}$$

This dictates the rate at which an air parcel cools as it rises (or warms as it sinks) in a stable, dry, atmosphere (the dry adiabatic lapse rate). If in a column of air a parcel aloft is warmer than predicted by this rate, then the column is stable. If instead a parcel of air aloft is cooler than predicted by this relation, then the column is unstable and warmer air from below will rise as the cooler air sinks (i.e. convection will take place).

We can now impose this lapse rate as a constraint in our simple model, which we will now refer to as a radiative convective equilibrium (RCE) model. That is, we are saying that if the temperature difference between the atmosphere aloft and at the surface is ever too great, the atmosphere in a column will mix vertically to relax the temperature profile to the dry adiabatic lapse rate, which is stable.

We can now reconsider how atmospheric opacity affects the temperature profiles of the surface and the atmosphere, depicted below. We can infer from this figure that convection consistently transfers energy from the surface to further up in the atmosphere. That is, the radiative convective equilibrium solution always has a warmer T_a and cooler T_s compared to the radiative equilibrium solution.

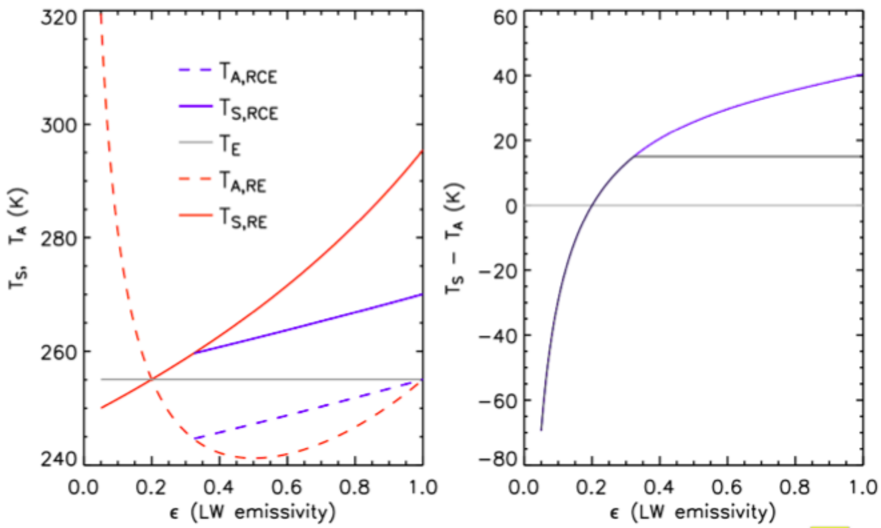

Figure 5: Figure credit: notes of Ron Miller, NASA GISS. The blue lines in the left panel indicate how the T_s and T_a profiles evolve once convection is introduced. The plateau of $T_s - T_a$ in the right panel is at the maximum allowable temperature difference before convection is triggered. The blue curve above the plateau is for the radiative equilibrium model only

When we add a greenhouse gas to the earth system, we change not only the temperature of the atmosphere and the surface, but also the vertical structure of temperature. To study this we need to move away from a two-layer model and into a continuous model. But to do so let's review and define a few concepts. We've already introduced the concept of an *effective emitting temperature* (T_e), but let's be a little more precise about what we mean by that and add to it the concept of an *effective emission height*.

In the absence of an atmosphere the earth simply obeyed the Stefan-Boltzmann law so that incoming radiation balanced outgoing radiation ($\frac{1}{4}S_0(1-\alpha) = \sigma T_e^4$), and all outgoing longwave radiation came from the surface. In this case the actual emitting temperature is the effective emitting temperature and the effective emission height is the surface. In a continuous atmosphere the effective emitting temperature is still the temperature needed to balance incoming solar radiation following the Stefan-Boltzman law (σT_e^4), but now the effective emission height is the height at which the atmospheric temperature matches this temperature. In reality each atmospheric layer contributes somewhat to the outgoing radiation, although the exact structure is complicated (as we will see).

So with these concepts in mind, let's explore how the greenhouse effect alters the surface temperature in an atmosphere in RCE. When we add a greenhouse gas to an atmosphere, we have increased the height at which radiation is absorbed and reemitted (the effective emission height). But because the lapse rate is constraining the rate at which temperature can change with height, and an energy balance is constraining the effective emitting temperature, the addition of a greenhouse gas must warm the surface (see figure below).

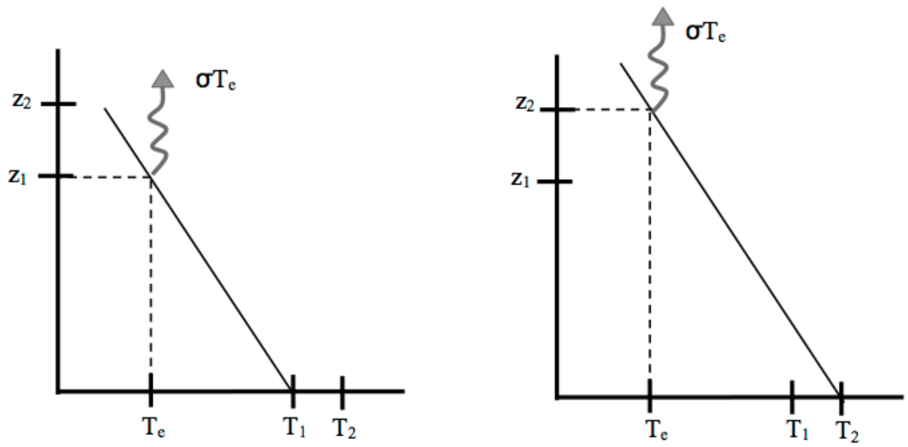

Figure 6: Changes in the effective emitting height with the addition of a greenhouse gas to an atmosphere in radiative convective equilibrium (RCE)

Now, at some height a parcel will become stable such that the temperature will follow the solution for radiative equilibrium (RE) rather than radiative convective equilibrium (RCE). So to construct a full temperature/height profile we will walk through a chain of logic aided by the figure below

1. In RE the net radiative fluxes balance (think of this as integrating the net radiation over all heights). Any modification to this profile must remain in balance

2. If we naively impose the lapse rate only to places where the temperature difference is greater than the lapse rate (dotted line below) we do not maintain a balance (i.e the area under the integral is drastically smaller).

3. To account for this, the level at which we impose the lapse rate must move upward (dashed line in the figure below) such that the area under the integral is (nearly) the same.

4. The reason it's not quite the same is because we need to include is the presence of turbulent fluxes (latent and sensible heating)

So our final adjustment from RE to RCE is to ensure that the change in the net radiation fluxes (i.e. difference in the area under the RE curve and the RCE curve in the figure below) is equal to the total energy flux associated with the turbulent fluxes.

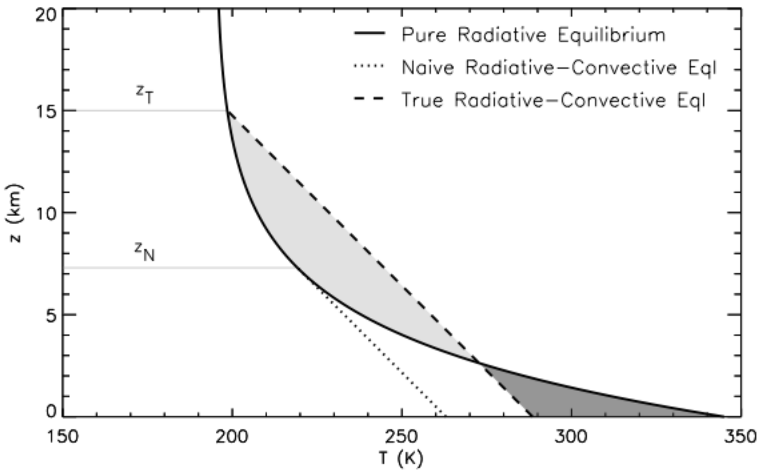

Figure 7: Figure credit: Ron Miller, NASA GISS

4.2 Patterns of warming and feedbacks

In this section we'll cover three dynamics that govern the predicted patterns of warming 1. arctic amplification (positive feedback), 2. the atmospheric lapse-rate feedback (negative feedback) and 3. the water vapor feedback (positive feedback).

4.2.1 Arctic amplification

As earth's surface warms, we expect that melting ice over either land or ocean will expose darker surfaces beneath. These darker surfaces will reflect less sunlight out to space, and therefore warm more quickly. This constitutes a positive feedback because as the arctic warms, we then expect the earth to absorb solar radiation more quickly and therefore warm faster, which will melt more ice. Satellite images of reflected solar radiation (left image) and changes in that reflection from 1979 to 2008 (right image) demonstrate that as the arctic warms and snow melts it is indeed reflecting less sunlight to space than it used to. This feedback is known as arctic amplification, and would lead us to believe that the poles will warm more quickly than the global mean temperature.

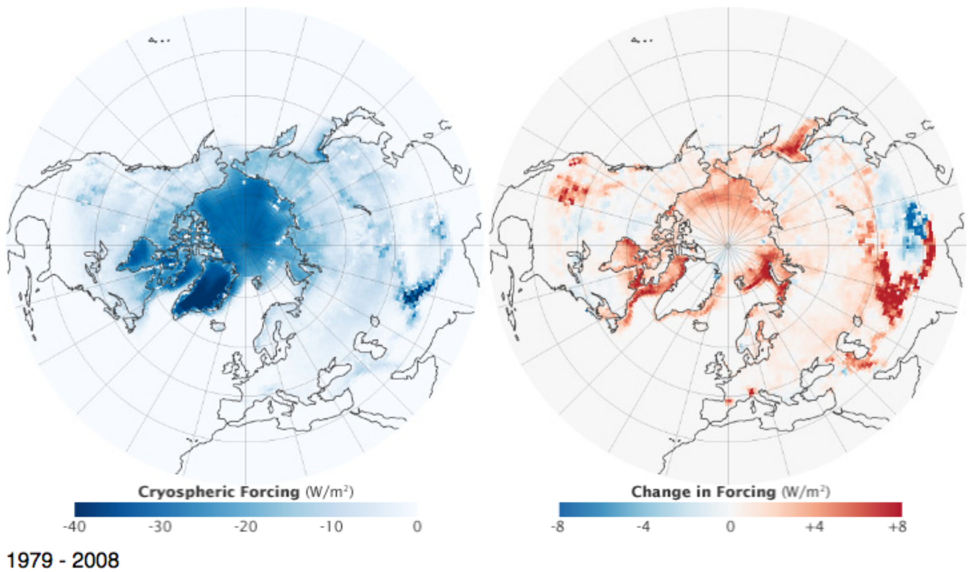

Figure 8: Figure credit: Ron Miller, NASA GISS via Flanner et al. (2011) https://www.nature.com/articles/ngeo1062

4.2.2 Water vapor feedback

The water vapor feedback refers to how the presence of water in the atmospheric column affects the expected rates of warming for a given forcing at the top of the atmosphere (ToA). This feedback exists because water is a greenhouse gas and warmer air holds more water. So as the earth warms, the air warms, and the concentration of greenhouse-gases (water vapor) increases in the atmosphere, further warming the earth.

We can also think of this using the framework we developed in the last section. Generally any ToA forcing will need to be balanced by increasing the outgoing longwave radiation (OLR) emitted to space, which follows the Stefan-Boltzman law ($OLR = \sigma T_s^4$). So we would expect the OLR to change nonlinearly (following T^4 instead). But that's not what we observe. In fact, the OLR changes more modestly than this because warmer air holds more water, which means that the atmospheric column is becoming more opaque to longwave radiation as it warms. As we discussed in the previous section, as the column warms longwave radiation moves to a higher level where the air is colder and therefore emits less efficiently (i.e. T is lower so OLR is lower). Saying that OLR increases more slowly than expected with surface warming is another way of saying the earth is re-emitting radiation less effectively, meaning that it is warming faster. This is the positive water vapor feedback.

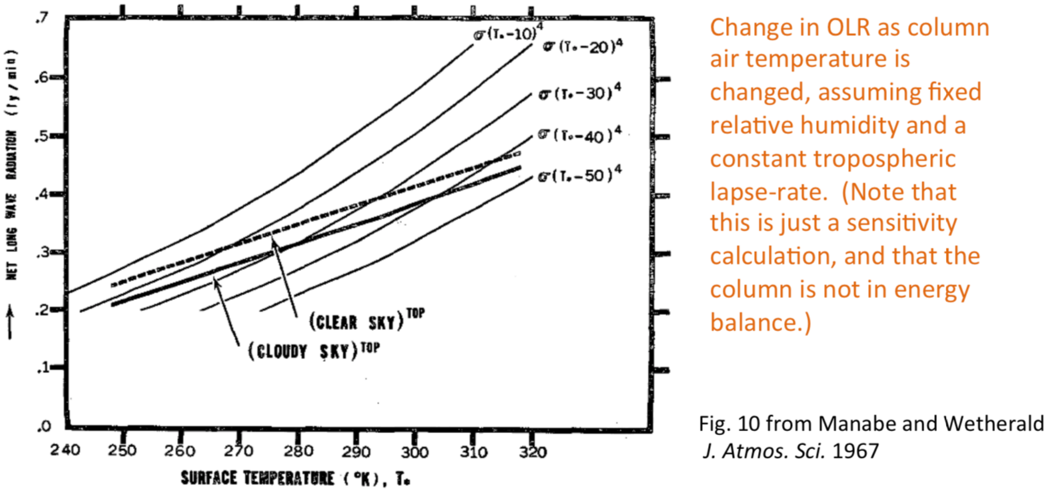

Figure 9: Figure credit: Ron Miller, NASA GISS

4.2.3 Lapse-rate feedback

The atmospheric lapse-rate feedback refers to how the lapse rate of the atmosphere changes with height as we warm the global atmosphere. Following from our discussion before, we can say that OLR adjusts to changes in ToA forcing by emitting from a greater height and therefore changing the upper-tropospheric temperature (where most OLR originates). In the tropics, where heat is mixed by deep convection, changing the upper-tropospheric temperature affects the temperature at lower levels. But because the lapse-rate changes with height (i.e. a moist adiabat), modest warming at the surface translates to much larger changes in temperature aloft. This is referred to as the negative lapse-rate feedback because a greater temperature increase aloft moderates the temperature increase at the surface. Increases in surface temperature lead to more water vapor that is transported via deep convection to the tropopause, and condensed out to release additional heat. Below is a schematic diagram of how we would expect the lapse-rate to change with surface warming.

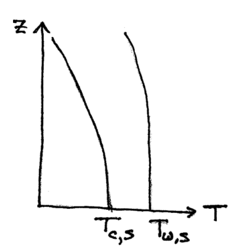

Figure 10: Figure credit: Ron Miller, NASA GISS

If we put all of these feedbacks together, and plot the results we can see that increased CO_2 will lead to faster warming in the upper tropical troposphere and the arctic.

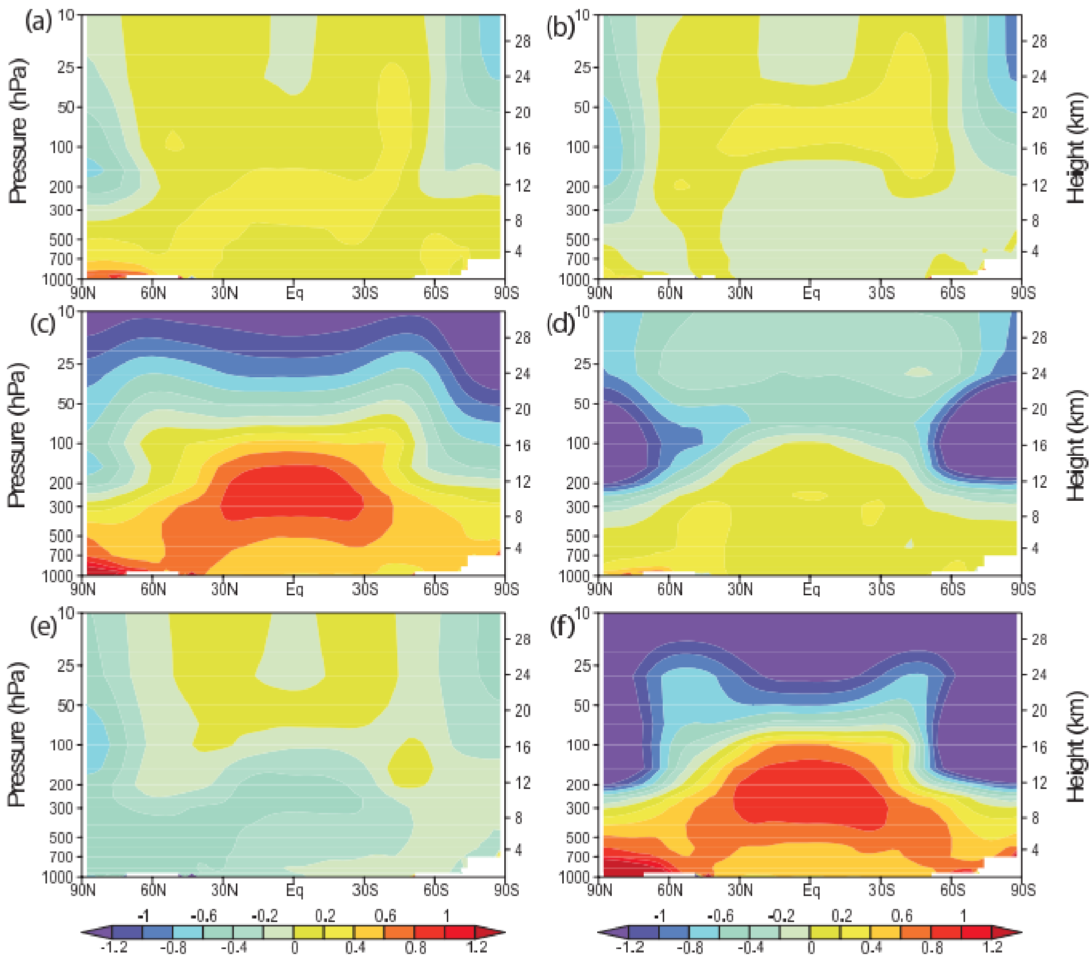

Figure 9.1. *Zonal mean atmospheric temperature change from 1890 to 1999 (°C per century) as simulated by the PCM model from (a) solar forcing, (b) volcanoes, (c) well-mixed greenhouse gases, (d) tropospheric and stratospheric ozone changes, (e) direct sulphate aerosol forcing and (f) the sum of all forcings. Plot is from 1,000 hPa to 10 hPa (shown on left scale) and from 0 km to 30 km (shown on right). See Appendix 9.C for additional information. Based on Santer et al. (2003a).*

Figure 11: Figure credit: IPCC AR4 WG1 Fig 9.1

5 Cloud Radiative Effects

We will next explore how clouds affect the radiative balance we just described. In general the impact that clouds have on the temperature of the atmosphere and the surface can be divided into two effects, the impact due to their albedo (clouds reflect incoming solar radiation back to space) and their radiative effects (i.e. they absorb and re-emit radiation depending on their temperature and liquid water content). The albedo effect is straight-forward: clouds are either transparent or they increase the albedo of the earth, which cools the surface. BUT even if clouds are reflective, this doesn't mean that the net impact of all clouds is to cool the surface, only that the impact due to their albedo is to cool the surface. So to calculate their net impact, let's move on to the radiative effects of clouds.

For this section we will classify clouds as either thin clouds (i.e. those with little liquid water) or thick clouds (those with lots of liquid water); as either high albedo (i.e. reflective) or low albedo (transparent to incoming solar radiation); and as cold clouds or warm clouds. In the tropics, for example, deep convection results in thick clouds that can reach high into the atmosphere (i.e. the tropopause). Because air cools as it lifts, the tops of clouds associated with deep convection are cold. So how will these clouds, which are cold at their top and thick (lots of liquid water), affect the radiative budget?

The first thing to note is that in the atmospheric window (i.e. where the surface of the earth most efficiently radiates to space) water vapor – indicated with H_2O in the below figure – is not an effective absorber, but liquid water is. So while humidity in the air (i.e. water vapor) is an effective greenhouse gas, it generally blocks emission at wavelengths outside of the atmospheric window (see below figure).

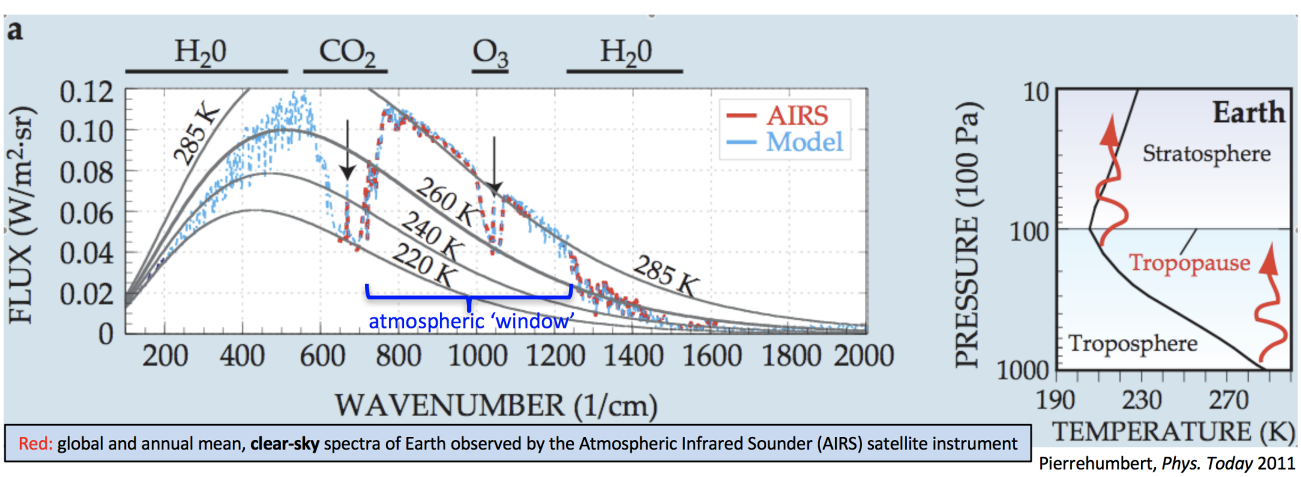

Figure 12: Figure credit: Ray Pierrehumbert, (Physics Today, 2011) via Ron Miller, NASA GISS

Liquid water, however, is absorptive in the atmospheric window. So a cloud that puts liquid water high into the atmosphere is equivalent to increasing the greenhouse effect (albeit temporarily). And as we saw in the previous section, an increased greenhouse effect will warm the surface (Fig 6).

We can also consider the effect of the temperature of the top of the cloud, which radiates to space. If there were no cloud, the surface of the earth would radiate directly to space according to the surface temperature of the earth. Because the cloud top is relatively cold compared to the surface it is less efficient at radiating to space, so the outgoing radiation from the atmosphere and the surface must increase, meaning the atmosphere and the surface will warm up.

So if we add these effects up **thick** (more water vapor, more GH effect, warms earths surface), **cold topped** (less efficient radiating to space, warms earths surface) clouds will warm the surface due to radiative effects, but because they reflect sunlight (they are **higher albedo**) they'll also cool the surface. These two effects tend to cancel one another.

On the other hand, we can consider low, warm, high albedo clouds. These low clouds tend to be a similar temperature to the earth's surface (i.e. warm) because the air doesn't have a chance to cool through adiabatic expansion that occurs in rising air parcels. They also tend to be thin (i.e. have less water vapor than clouds associated with deep convection). And they are high albedo (i.e. reflective). So let's now consider a thin, high albedo, warm topped cloud. A **thin** (not much water vapor, negligible GH effect), **warm topped** (similar temperature to earth surface, so nearly as efficient radiator), **high albedo** (reflects sunlight to space, cooling the surface) cloud will cool the surface of the earth because the albedo impact will be significant but the radiative impact will not.

Our final example will be of a cloud that is **thin** and **cold** (because it is high in the atmosphere) but is **low albedo** (transparent to incoming solar radiation). Just because a cloud is transparent to incoming solar radiation (which is shortwave), doesn't mean it is transparent to radiation emitted by the earth (which is longwave). This is essentially the same reason that CO_2 acts as a greenhouse gas even though it's not visible in the atmosphere. So the greenhouse forcing of these clouds acts to warm the surface. Because these clouds are high in the atmosphere, meaning they are cold, they don't efficiently radiate energy to space (again increasing the surface temperature). So **cold, thin, low albedo** clouds warm the surface because their radiative warming of the surface is large while their albedo is small and the cooling doesn't compensate.

Taken in total, the effect of clouds on the surface forcing is negative everywhere. So the effect of clouds is to cool the surface, and it will depend (to 0th order) on the amount and frequency of clouds. So even though the types of clouds found in the midlatitudes (i.e. cold, low) more effectively cool the surface, there are simply more clouds of all kinds in the tropics, meaning that the net surface forcing of clouds is largest in the tropics.

6 Aerosol effects

aerosols are small suspended particles (excluding clouds) that could be either liquid or solid.

Sources - Oceans are one of the most important means of aerosol formation. Bubble bursting and sea salt leads to aerosol formation. Smoke from forest fires is another major source of (organic) aerosols. Dust can also produce aerosols, provided the wind speed is great enough, and is, in fact, the largest contributor from the solid earth. Volcanos can inject particles into the troposphere (b/c of jets more effectively spread tropospheric aerosols in mid-latitudes than in the tropics) or into the stratosphere. Anthropogenic sources amount to about 20% that of natural sources.

Sinks - wet deposition (removal by precipitation) is episodic. Dry deposition (deposition onto vegetation and the land surface) occurs more slowly, but is continuous.

Radiative effect - Aerosols tend to have high albedo (with the exception of black carbon). This is why the global surface temperature drops following major volcanic eruptions (the direct effect). Note that the lower stratosphere may warm due to absorption of incident radiation by the aerosols following volcanic eruptions. The indirect effect refers to the effect of aerosol emissions for cloud formation, via production of condensation nuclei. The indirect effect was postulated following observations of whitening of stratus decks from exhaust plumes of ships. The indirect effect is generally thought to be negative, although it is highly uncertain (see below figure)

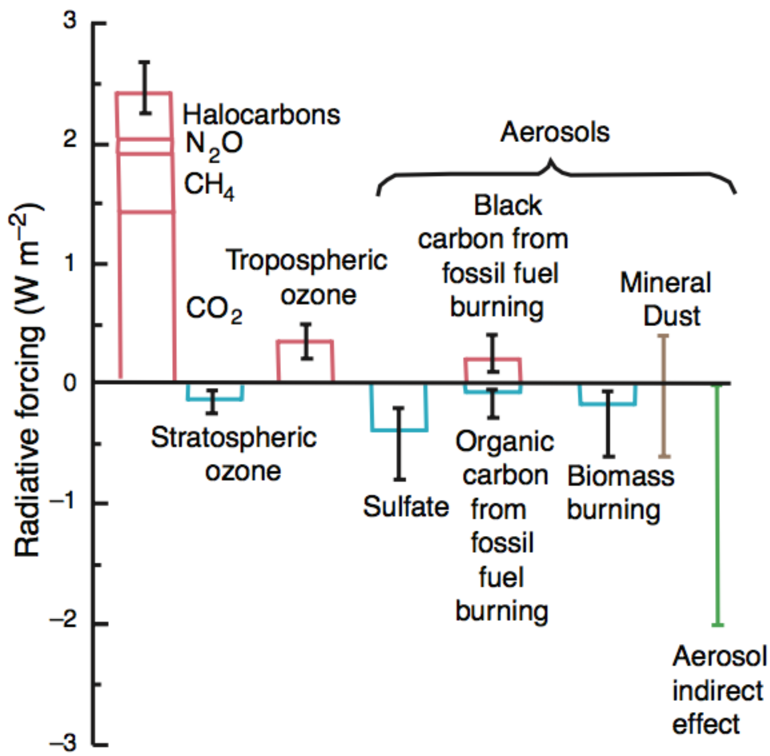

Fig. 10.42 Radiative forcing at the top of the atmosphere due to the human-induced incremental change in atmospheric concentrations of various species of trace gases and aerosols based on present concentrations minus pre-industrial concentrations. [Adapted from Intergovernmental Panel on Climate Change, *Climate Change 2001: The Scientific Basis*, Cambridge University Press, p. 8 (2001).]

Figure 13: Figure credit: (**?**)

7 Ozone (O_3)

Tropospheric O_3 is formed by lightning, or photochemical reactions using precursors from anthropogenic emissions of NO, CO and organic compounds.

Stratospheric O_3 plays an important role in the thermal structure of the atmosphere. Absorption of radiation by O_3 and longwave emission by CO_2 (and to a lesser extent H_2O is the dominant energy balance in the stratosphere. The absorption by O_3 (see Fig. 13) is the reason that there is a thermal inversion with height from the troposphere to the stratosphere.